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THE MATHEMATICS TEACHER

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THE AIMS OF TEACHING ALGEBRA AND HOW TO ATTAIN THEM.

By C. E. Biklé.

(Concluded.)

The work of many candidates is vitiated by the use of the continued inequation, as

$$x^8 - y^8 = (x^4 + y^4)(x^4 - y^4) = (x^2 + y^2)(x^2 - y^2) = (x + y)(x - y).$$

This kind of error is in part due to the careless work tolerated in arithmetic, as

$$2 \times 3 = 6 + 4 = 10 \div 5 = 2, \text{ and } 6\tfrac{4}{8} = \tfrac{1}{2}.$$

Adding and subtracting fractions are confused with clearing an equation of fractions.

Errors in numerical work abound, and many who are able to give the quadratic formula, and the general term in $(a + b)^n$ are unable to evaluate accurately.

I shall never forget the answers given by many candidates in 1902 to the questions "What do you understand by a^0? by a^{-1}? by $a^{\frac{1}{2}}$?" With confidence came the answers. "I understand that a^0 means a taken zero times as a factor, a^{-1} means a taken minus once as a factor, and $a^{\frac{1}{2}}$ means a taken one half a time as a factor." These pupils had evidently been well-drilled upon the definitions in the first part of the book, to the neglect, perhaps, of training in clear thinking. The problems leading to equations are still avoided.

MEANS AND REMEDIES.

A few days ago I received a postal card, informing me that a prominent western principal had had no failures in algebra in his school since he had adopted a certain text-book, although he had formerly had a large percentage of failures in that subject. This text-book, like other recent ones, has good points, but the presence of pictures and so-called real problems in the text-book will not relieve the teacher of the necessity of getting the pupils to think.

A live teacher who knows his subject and enjoys teaching it, and who is in the habit of consulting such helps as Chrystal, Oliver Wait and Jones, Fine, Smith and Young will be successful. His text-book may well contain problems taken from many books, directions for correlation of topics, and historical notes. A card-index of problems will be of use to him.

Just before I commenced teaching algebra it was my good fortune to take some work in mathematics with Dr. Durell. One reference book that I used was Oliver Wait and Jones. I believe that the ideas I got from the early pages of that book and from the "Drill-book in Algebra" by Professor Jones have been a great help to me.

To quote from the book first mentioned: "The symbols explained below constitute a symbolic language, a species of short-hand writing wherein numbers and their relations are more conveniently expressed than in the language of words. The signs stand for words and phrases, and generally have the same grammatical relations as the words and phrases themselves. The words may be restored at any time. The reader should constantly practice translating from one to the other till both are familiar. Among the many advantages of this symbolic langnage are clearness, brevity and generality of statement, the ability to mass directly under the eye and thus to bring before the mind all the steps in a long and intricate investigation, and the facility of tracing a number through all the changes it may undergo."

"Read in words the statements below, and point out verbs. conjunctions, etc.

$$(a+b)+(a-b)=2a$$

$$(a+b)-(a-b)=2b$$

$$(x + y)^2 - (x - y)^2 = 4xy$$

$$\frac{a + b}{2} + \frac{a - b}{2} = a$$

$$\frac{a + b}{2} - \frac{a - b}{2} = b$$

$$L - S = D$$

$$L - D = S$$

$$S + D = L.\text{”}$$

Emphasizing the sign of operation will help the pupil to see that $ba + ca = (b + c)a$,

$$s_1c + s_2c = (s_1 + s_2)c$$

$$\frac{Bh}{2} + \frac{bh}{2} = (B + b)\frac{h}{2}$$

$$\pi R^2 - \pi r^2 = (R^2 - r^2)\pi$$

$$\frac{\pi}{6}D^3 - \frac{\pi}{6}d^3 = (D^3 - d^3)\frac{\pi}{6}$$

Very legitimate work to secure interest in a beginning class is the following problem, taken from Ball's "Mathematical Recreations."

Think of a number.
Multiply it by 5.
Add 6.
Multiply by 4.
Add 9.
Multiply by 5.

Tell me the result, and I will tell you what number you had in mind at first.

The pupil wonders how the teacher is able to tell the number thought of, and a beginning of respect for algebra is secured when he sees that n, the number, is easily found from the result of the last operation, $100n + 165$. $\frac{2\pi R + 12}{2\pi} - R$ will help the pupil who is trying to find out whether he could slip the blade of his pen-knife between the circumference of the earth and a concentric circumference one foot longer.

The geometric pictures of $(a+b)^2$, $(a-b)^2$, $(a+b)(a-b)$, and such numerical applications as $(121)^2$, $(98)^2$, 125×115, will interest pupils and enlarge their grasp of numbers. I have found that pupils like to build a table of squares by using $(n+1)^2 = n^2 + 2n + 1$.

They are interested in applying factoring immediately to operations with fractions, and in solving equations of the second and higher degrees by using the factor theorem.

They enjoy reviewing their arithmetic problems from the algebraic point of view. The following illustration will serve. Given the times required by men separately to do a piece of work, find the time required when they work together. In the case of two men, with times t_1 and t_2, the result $\frac{t_1t_2}{t_1+t_2}$ may give the impression that the product of the times divided by the sum of the times is a formula including the cases in which more than two men are concerned. The result for three men, $\frac{t_1t_2t_3}{t_1t_2+t_1t_3+t_2t_3}$, removes this impression, and careful study of the equations from which the formula has been obtained brings an interpretation that includes the case in which any number of men is concerned.

By skilful questioning the teacher can help the pupils to think clearly, no matter what type of problems is under consideration. The pupil can usually find the equation which is hidden in the words of the problem if he sets about the work methodically. Very little illustrative material is necessary. The mystery of running water, of moving trains, of the courier riding to the head of the army, may be made clear without bringing into the class-room miniature rivers, railways or soldiers. The pupils must be taught to question themselves and to use diagrams as helps. The questioning is not unlike that used in geometry, the aim in each case being progress through the use of equations.

Pupils know that the time required to go in an automobile plus the time required to walk back will give the number of hours that have elapsed since the start, and most of them know the difference between rowing down stream and up stream.

While the pupils must work and check many problems having no content in themselves in order to acquire facility in operating with symbols, teachers may prevent the work from becoming mechanical by requiring the pupil to state the reason for each step. This takes time, but it increases the pupil's power of expression, and leads him to see that algebra is a science, and not a mere juggling with symbols.

Right here, it seems to me, is the secret of successful work in algebra. Teach algebra as a science, and classes will take greater interest in the subject and give greater returns in ground covered. Emphasize such points as the number of roots to be expected from an equation; show that $a^0 = 1$ is not a mystery, but a most reasonable convention.

Teach devices, and urge the pupils to use methods that require thought rather than manual labor. Keep in mind the mottoes. "Take a good look at the problem," and "Don't multiply until you have to," using such problems as

$$\frac{(a+b)^2(x+1)+(a+b)(x+1)+(x+1)}{a+b+1}=(a+b)^2+(a+b)+1$$

$$\frac{x+3}{x+1}-\frac{x+4}{x+2}=\frac{x-5}{x-3}-\frac{x-6}{x-4}$$

$$18(x+1)^2(x+2)^2=8(x-3)^2(x+1)^2$$

Above all, let the teacher himself know and love his subject. In many classes the work drags because the teacher is merely serving time. Let him know more than he is called upon to teach, and let him give his pupils some idea of the part which algebra is to play in their future work.

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MODERN TENDENCIES IN THE TEACHING OF ALGEBRA.*

By Fletcher Durell.

At present two movements are prominent, or at least noticeable, in the teaching of algebra: (1) The movement which would make the content or subject-matter less artificial and

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* Read at the meeting of the New York Section.